

## Degree 2 for $\mathcal{G}_2\mathcal{A}^{sv}$

September-05-13  
10:34 AM

$$D_1 = \text{Diagram } D_1$$

$$D_4 = \text{Diagram } D_4$$

$$D_7 = \text{Diagram } D_7$$

$$D_{10} = \text{Diagram } D_{10}$$

$$D_2 = \text{Diagram } D_2$$

$$D_5 = \text{Diagram } D_5$$

$$D_8 = \text{Diagram } D_8$$

$$D_{11} = \text{Diagram } D_{11}$$

$$D_3 = \text{Diagram } D_3$$

$$D_6 = \text{Diagram } D_6$$

$$D_9 = \text{Diagram } D_9$$

$$D_{12} = \text{Diagram } D_{12}$$

line. The ordering  $(ijk)$  becomes the relation  $D_3 + D_9 + D_3 = D_6 + D_3 + D_6$ . Likewise,  $(ikj) \mapsto D_6 + D_1 + D_{11} = D_3 + D_5 + D_1$ ,  $(jik) \mapsto D_{10} + D_2 + D_6 = D_2 + D_5 + D_3$ ,  $(jki) \mapsto D_4 + D_7 + D_1 = D_8 + D_1 + D_{11}$ ,  $(kij) \mapsto D_2 + D_7 + D_4 = D_{10} + D_2 + D_8$ , and  $(kji) \mapsto D_8 + D_4 + D_8 = D_4 + D_{12} + D_4$ . After some linear algebra, we find that  $\{D_1, D_2, D_6, D_8, D_9, D_{11}, D_{12}\}$  form a basis of  $\mathcal{G}_2\mathcal{A}^v(\uparrow)$ , and that the remaining diagrams reduce to the basis as follows:  $D_3 = 2D_6 - D_9$ ,  $D_4 = 2D_8 - D_{12}$ ,  $D_5 = D_9 + D_{11} - D_6$ ,  $D_7 = D_{11} + D_{12} - D_8$ , and  $D_{10} = D_{11}$ . In  $\mathcal{G}_2\mathcal{A}^{sv}(\uparrow)$  we have that  $D_5 = D_6$ ,  $D_7 = D_8$ , and  $D_9 = D_{10} = D_{11} = D_{12}$ ,

✓ Basis:  $\{1, 2, 6, 8, 9, 11, 12\}$

5 relations:

$$D_9 + D_{11} - D_6 = D_6 \Rightarrow D_9 + D_{11} = 2D_6$$

$$D_{11} + D_{12} - D_8 = D_8 \Rightarrow D_{11} + D_{12} = 2D_8$$

$$D_9 = D_6 = D_8 = D_{11}$$

5 basis:  $\{1, 2, 6, 8, 11\}$

$$D_3 = D_6 = D_5 = D_8 =$$

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